Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Linear Algebra
Subject Code : 5SC01LIA1 **Branch: M.Sc.(Mathematics)**
Semester : 1 **Date :19/03/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a) Define : i) Similar matrices ii) minimal polynomial of T (02)

b) Prove that $L(S)$ is subspace of V. (02)

c) Show that inner product space is linear. (02)

d) Define : Orthogonal Complement (01)

Q-2 **Attempt all questions** **(14)**

a) Let V and W be vector space over F of dimension m and n respectively. Then prove that $HOM(V,W)$ is of dimension mn over F. (07)

b) Let V be a finite dimensional vector space over F and W be subspace of V. Show that W is finite dimensional ,$\dim W \leq \dim V$ and $\dim V/W = \dim V - \dim W$. (07)

OR

Q-2 **Attempt all questions** **(14)**

a) Let V be a vector space over F then prove that V is isomorphic to a subspace of $\hat{\hat{V}}$. If V is finite dimensional then $\cong \hat{\hat{V}}$. (06)

b) State and prove Gram-Schmidth Orthonormalization process. (05)

c) Show that $W^{\perp}$ is subspace of V. (03)

Q-3 **Attempt all questions** **(14)**

a) If V is finite dimensional over F,and let $S,T \in A(V)$ and S be regular , then prove that $\lambda \in F$ is chatracteristic root of T if and only if it is a characteristic root of $S^{-1}TS$. (05)

b) Let V be finite dimensional over F and $T \in A(V)$. If $\lambda_1, \lambda_2, \dots \dots \dots . \lambda_k$ in F are distinct roots of T and $v_1, v_2, \dots \dots \dots . v_k$ are characteristic vector of T corresponding to $\lambda_1, \lambda_2, \dots \dots \dots . \lambda_k$ respectively. Then $v_1, v_2, \dots \dots \dots . v_k$ are linearly independent. (05)

c) Let V be finite dimensional over F and $T \in A(V)$ show that the number of characteristic root of T is atmost n^2. (04)

OR

Q-3 **Attempt all questions** **(14)**

	a)	Let V be a finite dimensional vector space over F and $S, T \in A(V)$.show that i) $rank(ST) \le rank(T)$ ii) $rank(TS) \le rank(T)$ iii) If S is regular then $rank(ST) = rank(TS) = rank(T)$	(05)
	b)	If V is finite dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V on to V.	(05)
	c)	Prove that $S \in A(V)$ is regular if and only if whenever $v_1, v_2, \dots \dots \dots . v_n \in V$ are linearly independent then $S(v_1), S(v_2), \dots \dots \dots . S(v_n)$ are also linearly independent.	(04)

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a)	Find the inertia of quadratic equation $x_1{}^2 - x_3{}^2 - 4x_1x_2 + 4x_2x_3 = 0$.	(02)
	b)	Prove that there do not exists $A, B \in M_n(F)$ such that $AB - BA = I$, where F is field with characteristic 0.	(02)
	c)	If $A, B \in M_n(F)$ then show that $tr\ (A + B) = tr(A) + tr(B)$	(02)
	d)	Define index of nilpotence.	(01)
Q-5		**Attempt all questions**	**(14)**
	a)	Let V be a finite dimensional vector space over F and $T \in A(V)$. If all the characteristic roots of T are in F then there is a basis of V with respect to which the matrix of T is upper triangular.	(07)
	b)	Let V be a finite dimensional vector space over F and $T \in A(V)$ be nilpotent then the invariants of T are unique.	(07)
		OR	
Q-5		**Attempt all questions**	(14)
	a)	Let V be a finite dimensional vector space over F and $T \in A(V)$. Suppose that $V = V_1 \oplus V_2$, where V_1 and V_2 are subspaces of V invariant under T. Let $T_1 = T\|_{V_1}$ and $T_2 = T\|_{V_2}$. if the minimal polynomial of T_1 over F is $p_1(x)$ while minimal polynomial of T_2 over F is $p_2(x)$. Then show that minimal polynomial of T over F is the least common multiple of $p_1(x)$ and $p_2(x)$.	(05)
	b)	Two nilpotent linear transformations are similar if and only if they have the same invariants.	(05)
	c)	Let V be a finite dimensional vector space over F and $T \in A(V)$. If all the characteristic roots of T are in F then T satisfies a polynomial of degree n over F.	(04)
Q-6		**Attempt all questions**	**(14)**
	a)	Let $A, B \in M_n(F)$,show that $\det(AB) = \det A \cdot \det B$.	(05)
	b)	State and prove Cramer's rule.	(05)
	c)	Prove that determinant of a matrix and its transpose are same.	(04)
		OR	
Q-6		**Attempt all Questions**	**(14)**
	a)	Prove that interchanging the two row of matrix changes the sign of its determinant.	(05)
	b)	Identify the surface given by $11x^2 + 6xy + 19y^2 = 80$. Also convert it to the standard form by finding the orthogonal matrix P.	(05)
	c)	If $\det A \neq 0$ then prove that A is regular.	(04)